Problem Session: One-Way ANOVA

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1. One-way ANOVA in brief

ANOVA tests whether the means of two or more groups differ. With three groups we would need three t-tests. At $\alpha = .05$ each, the overall risk of a Type I error is $1 - .95^3 = .143$. ANOVA does one test instead.

The scores in a study differ for two reasons. Scores differ *between* groups (treatment effect plus random differences) and *within* groups (random differences only). The F-ratio compares the two:

$$F = \frac{MS_{between}}{MS_{within}} = \frac{\text{treatment effect} + \text{random differences}}{\text{random differences}}$$

With no treatment effect, F is around 1. With an effect, F is much larger than 1.

Source	SS	df	MS
Between treatments	$SS_{total} - SS_{within}$ (or $\sum \frac{T^2}{n} - \frac{G^2}{N}$)	$k-1$	$SS_{between}/df_{between}$
Within treatments	$\sum SS$ inside each treatment	$N-k$	SS_{within}/df_{within}
Total	$\sum X^2 - \frac{G^2}{N}$	$N-1$	

- Effect size: $\eta^2 = \dfrac{SS_{between}}{SS_{total}}$ and $\omega^2 = \dfrac{SS_{between} - (df_{between})\,MS_{within}}{SS_{total} + MS_{within}}$
- A significant F means that at least one mean differs. Post hoc tests show which ones: Tukey's HSD (equal variances) or Games-Howell (unequal variances).
- Assumptions: independent observations, normality in each group (Shapiro-Wilk), equal variances (Levene). If they fail: Welch's ANOVA (unequal variances) or Kruskal-Wallis (non-normal data).

Warm-up. Three groups of children ($n = 4$ each) repeat nonwords. The scores are the number of errors.

Group A	Group B	Group C	
2 3 3 4	4 5 6 5	7 6 8 7	$N = 12$
$T = 12$	$T = 20$	$T = 28$	$G = 60$
$SS = 2$	$SS = 2$	$SS = 2$	$\sum X^2 = 338$

1.1 State H_0 and H_1.

1.2 Complete the table. The critical value for $df = (2, 9)$ and $\alpha = .05$ is 4.26. What is your decision?

Source	SS	df	MS	
Between treatments				$F =$
Within treatments				
Total				

1.3 Compute η^2 and ω^2.

1.4 Can we now say that Group A makes fewer errors than Group B? Why or why not?

2. One-way ANOVA assignment

Check your answers as we go through the five questions.

1. $SS_{total} =$ $SS_{within} =$ $SS_{between} =$
2. Number of treatments: Participants per treatment:
3. Three treatments, $n = 8$ each.

Source	SS	df	MS	
Between treatments				$F =$
Within treatments			2	
Total	62			

4. Study strategies (four groups, $n = 6$ each). $\alpha =$ $F_{critical}$ for $df = ()$:

Source	SS	df	MS	
Between treatments				$F =$
Within treatments				
Total				

Decision:

$\eta^2 =$ $\omega^2 =$

5. Homework by major (unequal samples). $\alpha =$ $F_{critical}$ for $df = ()$:

Source	SS	df	MS	
Between treatments				$F =$
Within treatments				
Total				

Decision:

3. R practice

3A. Log-transformed reaction times (`Pitt_Shoaf1.txt`). Reaction times are often right-skewed, so they are usually log-transformed. Do the ANOVA results hold for log RTs?

1. Create the new column: `ps1$log_rt <- log(ps1$rt)`
2. Check normality per group on `log_rt` (Shapiro-Wilk and Q-Q plots).
3. Fit the model: `log_model <- aov(log_rt ~ position, data = ps1)`
4. Run `summary()`, `eta_squared()` and `TukeyHSD()` on the model. Do the conclusions change?
5. How would you report the results of log-transformed data?

3B. VOT by consonant (`cherokeeVOT.txt`). Do velar stops (/k/) have a longer VOT than alveolar stops (/t/) in Cherokee? Note that the column name is `Consonant`, with a capital C.

1. Find the mean and *SD* of VOT by consonant, and make a boxplot.
2. Check normality per consonant and homogeneity of variance.
3. Run both `t.test()` and `wilcox.test(..., exact = FALSE)`. Calculate Cohen's d.

4. Which test is the proper one? Report its result in APA style.

3C. Treating the F1 data (`F1_data.txt`).

1. Make a boxplot of the pairwise differences (`f1_data$diff`).
2. Remove the pairs with a difference above 250 Hz.
3. Rerun the analyses with the proper test. Has anything changed?
4. Report the results in APA style.